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TECHNICAL NOTES

DESCRIPTION OF THE WETTING FRONT

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NOMENCLATURE

D	diffusivity
K	constant having the meaning of the moisture flux
t	time
u	moisture content
u_0	lower limit for $D \neq 0$
x	distance
X	position of the front.

THE HORIZONTAL unsaturated flow in porous media is usually described by the equation

$$\frac{\partial u}{\partial t} = \frac{\partial}{\partial x}\left(D(u)\frac{\partial u}{\partial x}\right), \tag{1}$$

where the diffusivity D is a function of the water content u. The commonly used functions $D(u)$ with $D(u) > 0$ for $u > 0$ do not give a proper description of the wetting front (the front is not 'sharp') [1]. To avoid this shortcoming, one can adopt the idea from the description of diffusion combined with an immobilizing process [2] and let $D(u)$ be zero for $u \in \langle 0, u_0 \rangle$, $u_0 > 0$. Equation (1) is completed by the condition

$$-\frac{\mathrm{d}X(t)}{\mathrm{d}t}u_0 = \lim_{x \to X(t)} D(u)\frac{\partial u}{\partial x}, \tag{2}$$

for this case.

It is not easy to solve equation (1) together with condition (2) for the general case. But a simple formula can be obtained for $X(t)$ in the steady-state approximation.

Equation (1) is solved with the conditions

$$u(x,t) = 0 \quad \text{for} \quad x \geqslant 0, \quad t = 0,$$
$$u(0,t) = 1 \quad \text{for} \quad t > 0,$$

and the moving boundary condition (2).

The steady-state approximation takes $u(x,t)$ in the form of the stationary solution on the $\langle 0, X(t) \rangle$ with the boundary conditions

$$\begin{aligned} u(X(t)) &= u_0, \\ u(0) &= 1. \end{aligned} \tag{3}$$

Equation (1) is reduced to

$$D(u)\frac{\partial u}{\partial x} = \text{const.} \tag{4}$$

in the stationary case.

The solution u of equation (4) satisfying conditions (3) is given by

$$\int_u^1 \mathrm{d}u'\, D(u') = Kx \tag{5a}$$

in implicit form

$$K = \frac{\int_{u_0}^1 \mathrm{d}u'\, D(u')}{X(t)}. \tag{5b}$$

As one can see, for the stationary case

$$-\lim_{x \to X(t)} D(u)\frac{\partial u}{\partial x} = K. \tag{6}$$

Equations (2) and (6) give

$$\frac{\mathrm{d}X(t)}{\mathrm{d}t}u_0 = \frac{\int_{u_0}^1 \mathrm{d}u'\, D(u')}{X(t)}, \tag{7}$$

which is the ordinary differential equation for $X(t)$. The solution satisfying the condition $X(0) = 0$ can be written in the form

$$X(t) = \sqrt{\left(\frac{2t\int_0^1 D(u)\,\mathrm{d}u}{u_0}\right)}. \tag{8}$$

This steady-state approximation formula can be improved, if one takes into account, that the flux near the boundary is less than that at $x = 0$. The improved form of equation (8) is

$$X(t) = \sqrt{\left(\frac{4t\int_0^1 D(u)\,\mathrm{d}u}{1+u_0}\right)}. \tag{9}$$

Introducing an example for

$$\begin{aligned} D(u) &= 0 \quad \text{for} \quad u \in \langle 0, \tfrac{1}{2} \rangle, \\ &= D_0(u - \tfrac{1}{2}) \quad \text{for} \quad u \in \langle \tfrac{1}{2}, 1 \rangle, \end{aligned}$$

one obtains

$$X(t) = \sqrt{\left(\frac{D_0 t}{3}\right)}, \tag{10}$$

and the moisture–distance curves have the shape given in Fig. 1. It is the typical shape of experimental results for water diffusion, e.g. in soil physics (it can be seen e.g. in refs. [1, 3]).

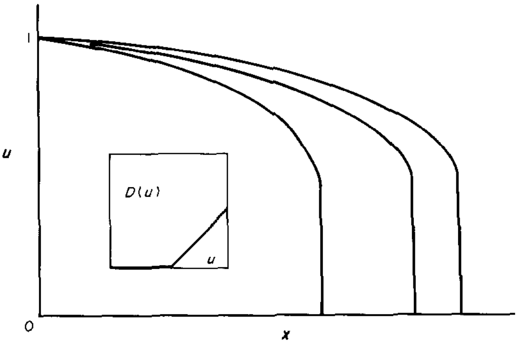

FIG. 1. Moisture distribution given by $D(u)$ for different times.

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THE EFFECT OF WALL SUCTION AND THERMOPHORESIS ON AEROSOL PARTICLE DEPOSITION FROM A LAMINAR BOUNDARY LAYER ON A FLAT PLATE

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NOMENCLATURE

$C_m, C_s, C_t, C_1, C_2, C_3$	constants in equation (2)
d	diameter
D	diffusion coefficient
f	dimensionless stream function, $\psi/(2u_e vx)^{1/2}$
g	dimensionless temperature, $(T-T_e)/(T_s-T_e)$
h_c	convective heat transfer coefficient
J	particle flux
k	thermal conductivity
Kn	Knudsen number
N	particle concentration
Pr	Prandtl number
Re	Reynolds number, $u_e x/v$
Sc	Schmidt number
St	Stanton number
T	temperature
u, v	streamwise and normal velocity components, respectively
V_T	thermophoretic velocity
x, y	streamwise and normal coordinates, respectively.

Greek symbols

δ_h	equivalent stagnant film thickness for heat transfer
ε	expansion parameter, $1/Sc_p$
ϕ	dimensionless particle concentration, N/N_e
η	similarity variable, $y(u_e/2vx)^{1/2}$
κ	thermophoresis parameter, $-V_T(T_s-T_e)/v(\partial T/\partial y)$
v	kinematic viscosity
ψ	stream function.

Subscripts

e	free stream
h	heat transfer
o	outer solution
p	particle
s	surface.

Superscripts

C	convection
D	Brownian diffusion
T	thermophoresis
$'$	differentiation with respect to η
$*$	limit of zero mass transfer rate.

1. INTRODUCTION

THERE are numerous technological and environmental problems which involve deposition of aerosol particles from a condensing vapor–gas mixture. Of current concern is deposition of radioactive aerosol particles inside a nuclear reactor containment following a hypothetical accident, which plays a critical role in reducing the amount of radioactivity that might be predicted to subsequently escape to the atmosphere. Deposition may occur on the walls of the pressure vessel, in pipes, or in bubbles rising in the pressure suppression pool. Mechanisms of deposition include Brownian diffusion, thermophoresis, diffusiophoresis, convection, inertial and sedimentation. Current engineering practice, e.g. computer codes such as HAARM-3 [1], PARDESIKO [2] and SUPRA [3], etc. is to calculate the deposition rates separately, and then to simply add them to obtain the total deposition rate. The purpose of this note is to examine the validity of such engineering calculation procedures.

The major situation of interest for nuclear reaction safety is deposition of particles from a steam-condensable gas mixture, where the gas might be air, hydrogen or a mixture of both. The flow may be forced or buoyancy driven; past a vertical or horizontal surface, laminar or turbulent, or could be a vortex flow inside a bubble. For all particle sizes the convective deposition associated with steam condensation plays an important role. For small particles, $d_p \lesssim 0.1$ μm, the diffusive mechanisms, i.e. Brownian diffusion, diffusiophoresis and thermophoresis can be important. For large particles, $d_p \gtrsim 1.0$ μm, inertial deposition and sedimentation can be important. In this note we consider only the small particles and study the coupling of convective and diffusive mechanisms of particle deposition.

The analysis of coupled deposition mechanisms has received some attention in the aerosol science and technology literature. Most of the published work deals with the coupling of Brownian diffusion and sedimentation, which is of limited interest in the current context. There has been some work on coupled thermophoresis and Brownian diffusion [4–6]. The most pertinent study is that of Hales *et al.* [7], who analyzed coupled convective and diffusive deposition of small aerosol particles in the laminar boundary layer associated with film condensation of steam from a stationary steam–air mixture on a vertical wall. The results, although correct and useful, do not clearly display the interactions between the various deposition mechanisms. We therefore chose to solve a simpler model problem in which transport and thermodynamic properties are taken to be constant. But in order to assume constant properties it is also necessary to ignore diffusiophoresis, since this phenomenon is related to the difference between the mass average and molar average velocities, which can only be calculated in a variable property analysis. The results of the analysis do, however, clearly display the interactions of the convective, Brownian diffusive and thermophoretic mechanisms of deposition, and allow an evaluation of approximations currently used in engineering practice.